(11 November 2011)

Abstracts of talks at JARCS4 2011

1. Jonathan Hillman (University of Sydney, Australia)
2-Knots with Solvable Groups

The closed 4-manifolds obtained by surgery on 2-knots with torsion-free, solvable groups have natural geometries of solvable Lie type. (There are two exceptions: the trivial knot and the remarkable Example 10 of R.H.Fox.) The knot exterior is obtained by deleting a regular neighbourhood of a simple closed curve representing a "weight orbit"; the orbit of a normal generator under the automorphisms of the group. We complete the topological classification of such 2-knots by finding the possible weight orbits, and using the geometries to decide when the exterior determines the knot.

2. Ruibin Zhang (University of Sydney, Australia)
Brauer tangle diagrams in classical invariant theory

We introduce a strict monoidal category, referred to as the category of Brauer tangles, and apply it to study the invariant theory of the orthogonal and symplectic groups. Full tensor functors are constructed from the category of Brauer tangles to the categories of tensor representations of the orthogonal and symplectic groups. This leads to a generalisation of the first and second fundamental theorems of invariant theory of these classical groups to a category theoretical setting. It in turn enables us to obtain new results on the algebraic structures of the endomorphism algebras of tensor powers of natural modules for orthogonal and symplectic groups. This is joint work with Gus Lehrer.

3. Laurentiu Paunescu (University of Sydney, Australia)
A'Campo Curvature Bumps Near A Singular Point

(joint with Satoshi Koike and Tzee-Char Kuo)

Let $f(x, y) \in \mathbb{R}\{x, y\}$ be a real analytic function-germ, $f(0, 0) = 0$. The level curves $f = c$, $0 < |c| < \epsilon$, have "bumps" near 0, as we all know.

Let us consider two simple examples:

$$f_2(x, y) = x^2 - y^3, \quad f_4(x, y) = x^4 - y^5.$$

We all know $f_2(x, y) = c$ attains maximum curvature when crossing the y-axis. However, a profound observation of N. A'Campo is that this is rather an isolated case. For example, the curvature of $f_4 = c$ is actually 0 on the y-axis; the maximum is attained instead as the level curve crosses $x = \pm ay^{4/3} + \cdots$, $a \neq 0$ a certain constant.

In this talk we explain this idea in the complex and the real cases, using the language of *(Newton-Puiseux) infinitesimals* and the notion of "*gradient canyon*". As we shall see, $x = \pm a y^{4/3} + \cdots$ are "*infinitesimals*", at each of which $f_4(x, y)$ has an "*A'Campo (curvature) bump*".

In the complex case, a different approach was taken by R. Langevin and E. Garcia Barroso - B. Teissier.

4. Adam Harris (University of New England, Australia)
Null deformations of the regular part of a cone singularity

Cone singularities have long been studied in complex analysis as those which arise from the collapsing of the zero section of a negative holomorphic line bundle, defined over a compact algebraic manifold. When infinitesimal Kodaira-Spencer deformations of complex structure are considered in the complement of the singular point, one may define "null deformations" as those with vanishing Kodaira-Spencer bracket. It may be somewhat surprising that such deformations are quite numerous, especially in the case of complex cones of dimension two, though they are generally distinct from the standard "stably embedded" deformations of the singularity. We will discuss the analytic characterization of these deformations within the versal parameter space.

5. Shuzo Izumi (Kinki University, Japan)
Projector onto a finite dimensional vector subspace of $\mathcal{O}_{n,a}$

Let $Z \subset \mathcal{O}_X(U)$ be a finite dimensional vector subspace of holomorphic functions on $U \subset \mathbb{C}^n$. Consider the set $Z_{\boldsymbol{b}}$ of germs of elements of Z at $\boldsymbol{b}$. It has the filtration by $Z_{\boldsymbol{b}}^{(k)} := Z_{\boldsymbol{b}} \cap \mathfrak{m}_{\boldsymbol{b}}^k$, where $\mathfrak{m}_{n,\boldsymbol{b}}$ denotes the maximal ideal of the local ring $\mathcal{O}_{n,\boldsymbol{b}} := \mathbb{C}\{\boldsymbol{t} - \boldsymbol{b}\}$. The initial form of $f(\boldsymbol{t})$ is denoted by $f_{\boldsymbol{b}}\!\downarrow$, expressed as a polynomial in Greek variables: $f_{\boldsymbol{b}}\!\downarrow \in \mathbb{C}[\boldsymbol{\tau}]$. Further they are identified with the signed derivative of the Dirac delta at $\boldsymbol{b}$. For example, $(\boldsymbol{t} - \boldsymbol{b})^{\boldsymbol{\nu}}_{\boldsymbol{b}}\!\downarrow = \boldsymbol{\tau}^{\boldsymbol{\nu}} = (-1)^{|\boldsymbol{\nu}|} \partial^{\boldsymbol{\nu}} \delta_{\boldsymbol{b}} / \partial \boldsymbol{t}^{\boldsymbol{\nu}}$. The linear span $Z_{\boldsymbol{b}}\!\downarrow := \bigoplus Z_{\boldsymbol{b}}^{(k)} / Z_{\boldsymbol{b}}^{(k+1)} =$ Span$\{f_{\boldsymbol{b}}\!\downarrow : f \in Z\}$ is the graded vector space associated to the filtration. Let $j_k Z$ denote the space of k-th jets of elements of Z.

Theorem I. Let us put $U_k := \{\boldsymbol{t} \in U : j_k Z$ is locally trivial around $\boldsymbol{t}\}$. Then $\bigcap_{k \in \mathbb{N}} U_k$ is analytically open in U. For any $\boldsymbol{b} \in \bigcap_{k \in \mathbb{N}} U_k$, $Z_{\boldsymbol{b}}\!\downarrow$ is *D-invariant*, i.e. invariant with respect $\partial / \partial \tau_i$.

The speaker have announced the three theorems below in 2009 in Nihon University. Later he found a gap in the proof of Theorem III. This gap is recovered by Theorem above. Tohru Morimoto suggested the speaker to use prolongation of PDEs annihilating Z for the proof of this theorem.

Let X be a complex submanifold of an open subset $\Omega \subset \mathbb{C}^m$. Let $P^d(X_{\boldsymbol{a}}) \subset \mathcal{O}_{X,\boldsymbol{a}}$ denote the vector subspace of all the polynomial functions on X of degree at most d. Take a local parametrisation $\boldsymbol{\Phi} : U_{\boldsymbol{b}} \longrightarrow X_{\boldsymbol{a}}$, the germ of the inverse of a local chart of X around $\boldsymbol{a}$. Let $\mathbb{C}[\boldsymbol{\Phi}]^d$ denote the space of pullbacks of $P^d(X_{\boldsymbol{a}})$ by $\boldsymbol{\Phi}$ and $\mathbb{C}[\boldsymbol{\Phi}]^d_{\boldsymbol{b}}\!\downarrow$ the graded space associated to it. The pushforward $D^{\varphi;d}_{X,\boldsymbol{a}} \subset \mathbb{C}[\boldsymbol{\xi}]$ of $\mathbb{C}[\boldsymbol{\Phi}]^d_{\boldsymbol{b}}\!\downarrow$ by $\boldsymbol{\Phi}$ has a sesquilinear product with the d-th jet space of $\mathcal{O}_{X,\boldsymbol{a}}$, which is non-degenerate on $D^{\varphi,d}_{X,\boldsymbol{a}} \times P^d(X_{\boldsymbol{a}})$. As the adjoint map of the inclusion $D^{\varphi,d}_{X,\boldsymbol{a}} \hookrightarrow D^{\varphi}_{X,\boldsymbol{a}} := \bigcup_{i\in\mathbb{N}} D^{\varphi,i}_{X,\boldsymbol{a}}$, we have a linear mapping $\mathcal{T}^{\varphi,d}_{\boldsymbol{a}} : \mathcal{O}_{X,\boldsymbol{a}} \longrightarrow P^d(X_{\boldsymbol{a}})$ such that

$$\left\langle p \,\middle|\, \mathcal{T}^{\varphi,d}_{\boldsymbol{a}}(f) \right\rangle = \langle p \,|\, f \rangle \ \text{ for any } (p,\, f) \in D^{\varphi,d}_{X,\boldsymbol{a}} \times \mathcal{O}_{X,\boldsymbol{a}}.$$

This $\mathcal{T}^{\varphi,d}_{\boldsymbol{a}}$ is noting but the *Taylor projector* of order d. If it is independent of the parametrisation $\boldsymbol{\Phi}$, we call $\boldsymbol{a}$ a *Taylorian point.*

Theorem II. The following conditons are equivalent for $\boldsymbol{a} \in X$.

1. $\boldsymbol{a}$ is a Taylorian point of order d.
2. $\mathcal{T}^{\varphi,d}_{\boldsymbol{a}}$ is a ring homomorphism and $P^d(X_{\boldsymbol{a}})$ has a natural structure of an Artinian ring.

Theorem III. Non-Taylorian points are contained in a thin analytic subset (the bad locus) of X.

These two theorems are proved by Bos-Calvi (2008a, 2008b) for plane algebraic curves.

We have defined the transcendency index $\alpha(X_{\boldsymbol{a}})$ of $X_{\boldsymbol{a}}$ (Iz. 1992, 1998). The following theorem implies that $X_{\boldsymbol{a}}$ is not highly transcendetal at generic points.

Theorem IV. If $\boldsymbol{a}$ is a Taylorian point for any $d \in \mathbb{N}$, we have $\alpha(X_{\boldsymbol{a}}) \leq m + \dim \overline{X} \leq m+n$. (The bad locus is a set of the first category in the sense of Baire and of Lebesgue measure 0 in X.)

Remark. A similar estimate is obtained for embeddings with Noetherian components using Gabrielov's multiplicity estimate (Gabrielov 1997) for Noetherian complete intersection (cf. Iz. 2003). There, the category is restrictive but the estimate is valid at all points of X.

6. Kiyoshi Takeuchi (Tsukuba University, Japan)
A-discriminants and A-hypergeometric functions

For finite sets A of lattice points Gelfand-Kapranov-Zelevinsky defined algebraic varieties called A-discriminants, which are natural generalizations of the classical discriminants. In this talk, we introduce a formula which expresses their degrees and dimensions in terms of the geometry of A. For this purpose we will give a combinatorial description of the Euler obstructions of non-normal toric varieties. In the second part, we introduce also A-hypergeometric functions whose singular sets are contained in the A-discriminants. Then we give their integral representations by using Hien's rapid decay homology cycles. Our formula can be considered as a higher-dimensional generalization of those for the classical Bessel and Airy functions. This is a joint work with Y. Matsui and A. Esterov.

7. Masahiro Shiota (Nagoya University, Japan)
Maps between analytic curve germs induced by blow-analytic homeomorphisms

Let $\tau : U \to V$ be a blow-analytic homeomorphism and $C(U)$ denote the analytic curve germs in U. Then τ induces a bijection from $C(U)$ to $C(V)$. We shows that it is a homeomorphism.

8. Jiro Sekiguchi (Tokyo University of Agriculture and Technology, Japan)

Holonomic systems related to free divisors and their solutions

K. Saito formulated the notion of a kind of hypersurfaces in a complex affince space C^n, so called a Saito free divisor or a free divisor in short. Moreover he also intoruced the notion of systems of uniformizations equations related with such a divisor. In a three dimensional case, I constructed many free divisors which have properties similar to discriminant sets of real reflection groups. The main purpose of my talk is to study solutions to holonomic systems with singularities along such free divisors.

9. Tomohiro Okuma (Yamagata University, Japan)

The embedding dimension of weighted homogeneous complex surface singularities

We consider a weighted homogeneous complex normal surface singularity. Then its link is a Seifert manifold determined by finite integers called Seifert invariants. We suppose that the link is a rational homology sphere. It is known that some fundamental analytic invariants (e.g., multiplicity, geometric genus) of such a singularity is topological, i.e., it can be determined from the link (or the Seifert invariants). However, in general, the embedding dimension is not a topological invariant. I will talk about a theorem when this invariant can be determined by the topology (joint work with Andras Nemethi).

10. Goo Ishikawa (Hokkaido University, Japan)

Openings of stable unfoldings

For a C^∞ map-germ $f : \mathbb{R}^n \to \mathbb{R}^m$, $(n \leq m)$, we introduce the notion of *openings* of f. An opening $F : \mathbb{R}^n \to \mathbb{R}^m \times \mathbb{R}^r$ of f, separates, via the projection $\pi_1 : \mathbb{R}^m \times \mathbb{R}^r \to \mathbb{R}^m$, the self-intersections of the original f, preserving the singularities of f. The notion of openings of f is different from the notion of unfoldings $\mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^m \times \mathbb{R}^k$: *Openings do not unfold the singularities.* For example, the *swallowtail* is an opening of the *Whitney's cusp* $\mathbb{R}^2 \to \mathbb{R}^2$ and the *open swallowtail* is an universal opening of them. Openings of map-germs appear as typical singularities in several problems of geometry and its applications. They are essential for the classification of those singularities. We will describe the openings of Morin singularities, namely, stable unfoldings of map-germs of corank 1. Moreover we will try to find the universal openings of stable unfoldings of map-germs of corank ≥ 2.

11. Osamu Saeki (Kyushu University, Japan)

Topology of definite fold singularities

It is known as the Reeb theorem that if a closed differentiable manifold admits a smooth function with only minima and maxima as its critical points, then the manifold is necessarily homeomorphic to the sphere. In this talk some generalizations of this theorem will be presented for smooth maps with only definite fold singularities into higher dimensional Euclidean spaces. Unlike the function case, the existence of such maps strongly affects the differentiable structure of the manifold. In the second part, elimination of definite fold singularities will be discussed in relation to broken Lefschetz fibrations and near-symplectic structures on 4-dimensional manifolds.

12. Masayuki Nishioka (Kyushu University, Japan)
Special generic maps between manifolds of certain dimensions

A special generic map is a smooth map of a closed n-dimensional manifold into $\mathbb{R}^p$ $(n > p)$ whose only allowed singularities are fold points of extremal index. Such a map was first defined by Burlet and de-Rham for $(n, p) = (3, 2)$ and later extended to general (n, p) by Porto, Furuya, Sakuma and Saeki. We give a topological restriction on the source manifold of special generic maps for $(n, p) = (2k + 1, k + 2)$ $(k > 1)$ and determine the diffeomorphism types of those 1-connected closed 5-manifold which admit special generic maps into $\mathbb{R}^4$.

13. Takao Akahori (Hyogo University, Japan)
On the complex analytic CR-Hamiltonian flows

Let M be a compact real hypersurface in a complex manifold N. We assume that our M is strongly pseudo convex. Then, related with the deformation theory of isolated singularities, the deformation theory of CR structures has been successfully developed(see [A2],[AGL]). And the notion of CR-Hamiltonian flows is found(see [AGL], [A4]). Especially, in [A4], we show that: if the generating funcion $g(t)$ is given (in this case, t is a real parameter), then we prove that the existence of the CR Hamiltonian flow $f_{g(t)}$, associaed with $g(t)$, which depends on t, $C^\infty - ly$. Here we report that: if $g(s)$ depends on s, complex analytically, then also we have the CR Hamiltonian flow $f_{g(s)}$, which depends on s, complex analytically.

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14. Mahito Kobayashi (Akita University, Japan)

On the bubbling surgery of a smooth map

A stable map between manifolds $f : M \to N$ can be considered a tool to understand M by comparing it with a better known, lower dimensional one N. But it is generally difficult to obtain a stable map between given manifolds in an explicit way. We introduce a practical tool to modify a stable map f above to another. It acts on M as a surgery while leaves N unchanged, and adds a co-dimension 1-sphere to the locus of critical values.

As an application, singular fibrations between spheres $S^n \to S^k$ are obtained (k is one of $2, 4$ or 8 and n is any integer with $n > 2k-1$) which are stable and possess the simplest locus of critical values among the stable maps between these spheres.

As another application, a geometric representation of manifold named a portrait is introduced by using a stable map and given two in-finiteness properties about it.

15. Mutsuo Oka (Tokyo University of Science, Japan)

Varchenko-type theorem for mixed functions

A mixed function is called of strongly polar weighted face type if evry face function f_P is strongly polar weighted homogeneous. We generalize the result of Varchenko for the zeta function to such a mixed function which is non-degenerate.

16. Toshizumi Fukui (Saitama University, Japan)

Motivic zeta functions for real polynomial germ and real weighted homogeneous polynomials

An expression of the motivic zeta function for a real polynomial function in terms of Newton polyhedron will be given. An application to weighted homogeneous polynomials will be discussed. This is a joint work with Goulwen Fichou.

17. Shinichi Tajima (Tsukuba University, Japan), Yayoi Nakamura (Kinki University, Japan)

μ-constant deformations, parametric local cohomology and Tjurina numbers

We derive a new effective method to study μ-constant deformations of semi-quasihomogeneous isolated hypersurface singularities. The resulting algorithm which is free from Kodaira-Spencer map computes Tjurina stratifications of deformation spaces and parametric standard bases of ideal quotients in question. The main ideas of this new approach are the use of parametric algebraic local cohomology and Grothendieck local duality theorem in computations.