List of Publication

Journal articles.

[1] (with T.-C. Kuo and L. Paunescu) *A'Campo curvature bumps and the Dirac phenomenon near a singular point*, Proc. London Math. Soc. **111** (2015), 717–748.

[2] (with L. Paunescu) (*SSP*) *geometry with directional homeomorphisms*, J. of Singularities **13** (2015), 169–178.

[3] (with L. Paunescu) *On the geometry of sets satisfying the sequence selection property*, J. Math. Soc. Japan **67** (2015), 721–751.

[4] (with Ta Lê Loi, L. Paunescu and M. Shiota) *Directional properties of sets definable in o-minimal structures*, Ann. Inst. Fourier **63** (2013), 2017–2047.

[5] (with A. Parusiński) *Equivalence relations for two variable real analytic function germs*, J. Math. Soc. Japan, **65** (2013), 237–276.

[6] *Finiteness theorem for Blow-semialgebraic triviality of a family of three-dimensional algebraic sets*, Proc. London Math. Soc. **165** (2012), 506–540.

[7] (with Tzee-Char Kuo and Laurentiu Paunescu) *A study of curvature using infinitesimals*, Proc. Japan Acad. Ser. A. Math. Sci. **88** No. 5 (2012), 70–74.

[8] (with A. Parusiński) *Some questions on the Fukui numerical set for complex function germs*, Demonstratio Mathematica **XLIII** (2010), 285–302.

[9] (with A. Parusiński) *Blow-analytic equivalence of two variable real analytic function germs*, J. Algebraic Geometry **19** (2010), 439–472.

[10] (with L. Paunescu) *The directional dimension of subanalytic sets is invariant under bi-Lipschitz homeomorphisms*, Ann. Inst. Fourier **59** (2009), 2445–2467.

[11] (with M. Shiota) *Non-smooth points set of fibres of a Nash mapping*, J. Math. Soc. Japan, **59** (2007), 953–969.

[12] *Finiteness theorems on Blow-Nash triviality for a family of zero-sets of Nash mappings*, Banach Center Publications, **65** (2004), 135–149.

[13] (with A. Parusiński) *Motivic-type invariants of blow-analytic equivalence*, Ann. Inst. Fourier, **53** (2003), 2061–2104.

[14] (with S. Izumi and T.C. Kuo) *Computations and stability of the Fukui invariant*, Compositio Math., **130** (2002), 49–73.

[15] *Nash trivial simultaneous resolution for a family of zero-sets of Nash mappings*, Math. Zeitschrift, **234** (2000), 313–338.

[16] *Blow-analytic SV-sufficiency does not always imply Blow-analytic sufficiency*, Hokkaido Math. J., **28** (1999), 385–392.

[17] (with T. Fukui and M. Shiota) *Modified Nash triviality of a family of zero-sets of real polynomial mappings*, Ann. Inst. Fourier, **48** (1998), 1395–1440.

[18] (with K. Bekka) *The Kuo condition, an inequality of Thom's type and* (C)*-regularity*, Topology, **37** (1998), 45–62.

[19] *Modified Nash triviality theorem for a family of zero-sets of weighted homogeneous*

polynomial mappings, J. Math. Soc. Japan, **49** (1997), 617–631.
[20] *Modified Nash triviality of a family of zero-sets of weighted homogeneous polynomial mappings*, Kodai Math. J., **17** (1994), 432–437.
[21] *On strong C^0-equivalence of real analytic functions*, J. Math. Soc. Japan, **45** (1993), 313–320.
[22] *Notes on C^0-sufficiency of quasijets*, J. Math. Soc. Japan, **42** (1990), 265–275.
[23] *C^0-sufficiency of jets via blowing-up*, J. Math. Kyoto Univ., **28** (1988), 605–614.
[24] (with G. Ishikawa and M. Shiota) *Critical value sets of generic mappings*, Pacific J. Math., **114** (1984), 165–174.
[25] *On condition (a_f) of a stratified mapping*, Ann. Inst. Fourier, **33** (1983), 177–184.
[26] *On v-sufficiency and $(\overline{h})$-regularity*, Publ. Res. Inst. Math. Sci., **17** (1981), 565–575.
[27] *A remark on sufficiency of jets*, Math. Japonica, **25** (1980), 671–676.
[28] (with W. Kucharz) *Sur les réalisations de jets non suffisants*, C. R. Acad. Sci., **288** (1979), 457–459.

Refereed conference papers.

[C1] (with Tzee-Char Kuo and L. Paunescu) *Non concentration of curvature near singular points of two variable analytic functions*, Topics on Real and Complex Singularities (Proceedings of the 4-th Japanese-Australian Workshop (JARCS4), Kobe, 2011), World Scientific, pp. 115–140, 2014.
[C2] (with K. Bekka and T. Fukui) *On the realisation of a map of certain class as a desingularisation map*, Proceedings of the Australian-Japanese Workshop on Real and Complex Singularities, World Scientific, 2007, pp. 33–45.
[C3] (with T. Fukui and T.C. Kuo) *Blow-analytic equisingularities, properties, problems and progress*, Real analytic and algebraic singularities, Pitman Research Notes in Mathematics Series, **381** (1998), pp. 8–9.
[C4] *A problem on partition of analytic functions*, Stratifications, Singularities and Differential Equations I, Travaux en Cours **54**, Hermann (1997), pp. 63–81.

Book chapters.

[B1] *Invariants of real analytic singularities* (in Japanese), The World of Singularities, Have Fun with Mathematics (Autumn 2005), pp. 80–93.

Conference papers (non-refereed).

(Research Institute of Mathematical Sciences, Kyoto University, Japan)

[R1] *On finiteness of semialgebraic types for a family of Nash maps defined on a Nash surface*, RIMS Kokyuroku, **1948** (2015), 1–5.
[R2] *Generalization of directional properties on the real field to the case of general real closed field*, RIMS Kokyuroku, **1707** (2010), 67–75.
[R3] *o-minimal category and desingularisation theorem*, RIMS Kokyuroku, **1540** (2007), 99–106.

[R4] (with A. Parusiński) *Motivic zeta functions for real analytic functions and their applications to blow-analytic equisingularity problems*, RIMS Kokyuroku, **1374** (2004), 52–78.
[R5] *The Briançon-Speder and Oka families are not biLipschitz trivial*, RIMS Kokyuroku, **1328** (2003), 165–173.
[R6] *A problem on blow-analytic sufficiency of jets*, RIMS Kokyuroku, **1111** (1999), 165–167.
[R7] (with K. Bekka) *The Kuo condition, Thom's type inequality and (C)-regularity*, RIMS Kokyuroku, **952** (1996), 41–49.
[R8] *On finite modified Nash V-determinacy of polynomial map-germs*, RIMS Kokyuroku, **926** (1995), 113–117.
[R9] *Strong C^0 triviality of family of weighted homogeneous polynomials*, RIMS Kokyuroku, **815** (1992), 110–122.
[R10] *Notes on C^0 determinacy of analytic functions related to weights*, RIMS Kokyuroku, **690** (1989), 98–111.
[R11] *On C^0-equivalence of functions*, RIMS Kokyuroku, **619** (1987), 114–134.
[R12] *C^0-sufficiency via blowing-up*, RIMS Kokyuroku, **550** (1985), 112–122.
[R13] *Zero-sets of real homogeneous polynomials and their coefficient space*, RIMS Kokyuroku, **493** (1983), 39–47.
[R14] *On v-sufficiency and (h)-regularity*, RIMS Kokyuroku, **403** (1980), 96–103.

(1) Singularity theory of differentiable maps.

The pioneer of the singularity theory of differentiable maps between differentiable manifolds is H. Whitney. He started to work on the structural stability problem of differentiable maps which is the most important problem in this field. In fact, he established a structural stability theorem in the plane-to-plane case, and introduced and thought up many important notions and techniques to establish the structural stability in the general case. There are many terminologies named Whitney in this field, e.g. the Whitney topology of the set of differentiable maps, the Whitney stratification with the Whitney regularity conditions, the Whitney trick, the Whitney functions and so on.

After Whitney, R. Thom gave the biggest contribution to the singularity theory of differentiable maps. In order to establish the structural stability theory, Thom presented a big framework with many important new ideas, tools and notions. Thom's First and Second Isotopy Lemmas are very famous tools to show the topological triviality of a family of sets or maps, and to establish the topological structural stability theorem. The First Isotopy Lemma works on the Whitney stratification, and the Second one works on the Whitney stratification with the Thom (a_f) regularity condition.

One of important notions introduced by Thom is "sufficiency of jets" or "finite determinacy". Basically, the notion of sufficiency of jets is a local one. Nevertheless, when we started to study the real singularity theory, many of us used to be attracted by the problem to determine the order of sufficiency of jets (or finite determinacy) using the power of ideals or the Lojasiewicz exponent.

The most popular topics in this field are to give a criterion or a charcterisation for sufficiency of jets. The papers [22], [23], [26], [C4] and [R11] are devoted to the topics. In fact, I gave criteria for C^0 sufficiency of weighted jets and C^0 sufficiencey of jets via blowing up in [22] and [23], respectively. On the other hand, I gave characterisations of V sufficiency of jets and C^0 sufficiency of jets with regularity conditions in [26] amd [R11], respectively. In addition, I discussed in [C4] a partition problem of an analytic function using the results on sufficiency of jets.

In a joint paper with W. Kucharz [28], I gave a counterexample to Thom's conjecture on the realisations of non-sufficient jets. On the other hand, I proved in [27] that R-C^i wildness is equivalent to R-C^0 wildness for $i = 1, \cdots$. The existence of "wild jet" implies unstability in a certain sense.

In [16] I discussed the relationship between Blow-analytic SV-sufficiency and Blow-analytic sufficiency in real analytic function germs. On the other hand, R. Thom and T.-C. Kuo had given different criteria for C^0 sufficiency of jets and V sufficiency of jets. I proved with K. Bekka in [18] that their criteria are equivalent conditions in the functrion case, and we generalised the result to the mapping case.

Concerning the stratification theory, I have written two papers. Thom condition (a_f) has a geometric contents. I proved in [25] that (a_f) and the geometric contents are equivalent.

When we consider the stratification of a generic differentiable map, the critical value set becomes important. From this viewpoint, I proved with G. Ishikawa and M. Shiota in [24] that the critical value set of a generic polynomial map is the main part of some algebraic set, and a similar result holds also in the analytic category.

(2) Equisingularity problem of algebraic and analytic singularities.

The equisingularity problem of complex algebraic and analytic singularities has a long history. In fact, it has been studied more than a century. The first modern approach to the problem was made by O. Zariski. Then numerous remarkable theorems have been established by Zariski and many succeeding algebraic geometers and complex singularitists.

For the real case, however, the history has been quite short, and there are only a few outstanding results. One of them is the blow-analytic theory initiated by T.-C. Kuo. He established the foundation of the theory, in particular, a finite classification theorem on blow-analytic equivalence for a subanalytic family of real analytic function germs with isolated singularities. Blow-analyticity is the equisingularity of real analytic function germs.

In Kuo's early works on blow-analycity, he announced that blow-analytic equivalence preserves the contact order of analytic arcs. But I showed in [21] that it is not valid. More precisely, I showed that if we regard the Briançon-Speder family as a family of real polynomial functions, then it is blow-analytically trivial as a family of functions but the family of zero-sets is not trivialised by any homeomorphism preserving the contact order of analytic arcs. We call such homeomorphism "strong homeomorphism". In [R9] I discussed characterisations of strong C^0 equivalence for weighted homogeneous polynomials.

After the above works, I have been interested also in introducing and analysing invariants of real analytic singularities. I gave several formulae with S. Izumi and Kuo in [14] to compute the Fukui blow-analytic invariants. With A. Parusiński, I introduced some motivic-type invariants for the blow-analytic equivalence relation ([13], [R4]). The Fukui blow-analytic invariants and our motivic-type invariants are sufficient to give a blow-analytic classification of two variable real Brieskorn polynomials. On the other hand, there exist two variable real polynomial functions with isolated singularities which are not blow-analytically equivalent, but whose Fukui invariants and also our motivic zeta functions coincide. Then we gave in [9] a complete classification of two variable real analytic fucntion germs by real tree-model and real resolution graph. Using this result, we proved in [5] that C^1 equivalence implies blow-analytic one for two variable real analytic function germs, which is a negative result to Kuo's conjecture on blow-analytic equivalence and C^r equivalence. Apart from these works we discussed in [8] whether the Fukui invariant is a topological invariant or not in the complex case.

I have written the first survey [C3] on the blow-analytic theory with Kuo and Fukui, and a survey [B1] on invariants of real analytic singularities.

Concentration of curvature has been studied by complex singularitists as an equisingularity of complex analytic function germs. I worked with T.-C. Kuo and L. Paunescu on this topic. In a family of level curves of a two variable analytic function germ, bumps can appear near the singularity where the Gaussian curvature takes the maximum. In [1] (announced in [7]) we characterised where such bumps appear, using the Newton-Puiseux infinitesimals and the notion of gradiant canyon. On the other hand, we studied in [C1] the phenomenon of non-concentration of curvature of level curves of two variable analytic function germs. We characterised it in terms of tree models and topological types in the complex case. In the real case we discussed the relationship between the phenomenon and real tree models (or blow-analytic types).

Motivated by the desingularisation theorem of Hironaka and Bierstone-Milman, the Nash manifold theory of M. Shiota and the blow-analytic theory of T.-C. Kuo, I started to work on an equisingularity problem of real algebraic singularities. In the past 20 years, I have laid the foundation of the Blow-Nash Theory for real algebraic singularities, and have established a number of Blow-Nash deformation theorems and finite classification theorems for a family of the zero-sets of Nash maps. We call the zero-set of a Nash map a "Nash set" after this. The aforementioned results on equisingularity of analytic singularities are local ones, but our Blow-Nash theory is basically a global one.

I first proved in [19] a Blow-Nash deformation theorem (modified Nash tiriviality theorem) for a family of zero-sets of weighted homogeneous polynomial maps, and a Nash Isotopy Lemma for a pair of a Nash manifold and its Nash submanifold. Then I proved with T. Fukui and M. Shiota two types of Blow-Nash deformation theorems, and a finite classification theorem for a Nash family of Nash set-germs with isolated singularities ([17]). In order to show these results, we proved also a Nash Isotopy Lemma in the normal crossing case. Using this Isotopy Lemma, I proved in [15] a finiteness theorem on the existence of Nash trivial simultaneous resolution for a Nash family of Nash sets defined on a compact Nash manifold, and a finite classification theorem on Blow-semialgebraic triviality for a family of compact Nash surfaces. In this paper I am treating the global and non-isolated singularity case. I wrote a survey [12] on the above finiteness results on Blow-Nash triviality for a Nash family of Nash sets.

Subsequently I have proved a finiteness theorem on Blow-semialgebraic triviality for a family of 3 dimensional Nash sets with non-isolated singularities defined over a non-compact Nash manifold, and a Nash Isotopy Lemma in the normal crossing case without the assumption of properness which is applicable to the non-compact case. As a result, I improved in [6] all the former results as mentioned above. Related to the above finiteness results, I have proved the representation theorem of a polynomial map as a resolution map jointly with K. Bekka and T. Fukui ([C2]). It is well-known that topological moduli appear in the Nakai family. This family is represented by simultaneous resolution maps of a family of 4-dimensional algebraic sets. In addition, generalising a result shown in [6], I proved with Shiota a finiteness theorem on the number of semialgebraic types appearing in a family of Nash maps defined over a Nash surface with isolated singularities, and constructed a family of polynomial maps defined over an algebraic surface with non-isolated singularities in which topological moduli appear. The results were announced in [R1].

On the other hand, from the viewpoint of the global equisingularity of algebraic singularities, I discussed the structure of the non-smooth points set of fibres of a Nash map with Shiota ([11]).

(3) Directional properties of a singular space with Lipschitz transformations.

H. Whitney introduced the notion of the Whitney stratification endowed with some regularity conditions in order to show that an algebraic set or an analytic set has a locally, topologically trivial structure. Then S. Lojasiewicz introduced the notions of a semialgebraic set and a semianalytic set, and showed that they admit a Whitney stratification. In addition, H. Hironaka introduced the notion of a subanalytic set, and showed that it admits a Whitney stratification. The Whitney regularities are directional conditions for a singular space.

I have started to work with L. Paunescu on directional properties of a singular space from a different viewpoint from a series of works initiated by Whitney. Our works are closely related to the introduction of Lipschitz invariants.

M. Oka is constructing a family of 3 variable complex polynomial functions which is μ-constant but not μ^*-constant. It is a different type from the Briançon-Speder family. Using some directional property, I proved in [R5] that if we regard the Oka family as a family of real polynomials, then it is topologically trivial but not Lipschitz trivial. Then, generalising the directional property and introducing the new directional property called (SSP), I proved with Paunescu in [10] that the dimension of the common direction set of subanalytic sets is a Lipschitz invariant. In [4] I generalised this result with Ta Lê Loi, Paunescu and M. Shiota to the case of definable sets in an o-minimal structure on any real closed field.

The (SSP) category is much wider than the subanalytic one. A subanalytic set, a definable set in an o-minimal structure and the cone of any set satisfy condition (SSP). Condition (SSP) is not a Lipschitz invariant, but we can introduce various Lipschitz invariants using (SSP). I proved with Paunescu in [3] a lot of directional properties with

(SSP) and Lipschitz transformations in order to develop the geometry of sets satisfying condition (SSP), e.g. (SSP) structure preserving theorem, a transversality theorem in the singular case and so on. In addition, we generalised some important properties shown in [3] to the case of directional homeomorphisms ([2]).